Constraints on active-sterile neutrinos and cosmological bounds

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- Plan of the talk
- Introduction
  1. Neutrino oscillations
  2. Active-sterile neutrino mixing
  3. Neutrinos and primordial abundances
  4. Signal to noise ratios for highly energetic neutrinos
- Conclusions

Introduction

- Neutrinos do have a mass and they oscillates between flavor states.
- The information on primordial abundances (BBN) may be used to constraint oscillation parameters
- The mixing between sterile and active neutrinos may also be fixed from BBN observables
- Neutrinos from distant and very energetic objects, like micro quasars, may be measured (signal to noise ratios for these events need to be calculated)
- ANDES mega-detector?

Neutrino oscillations

- neutrino oscillations

$|\nu_1(t)>$, $|\nu_2(t)>$, $|\nu_3(t)>$ are three mass eigenstates at time t, then

$$\imath \frac{d}{dt} \begin{pmatrix} |\nu_1(t)> \\ |\nu_2(t)> \\ |\nu_3(t)> \end{pmatrix} = H^{(m)} \begin{pmatrix} |\nu_1(t)> \\ |\nu_2(t)> \\ |\nu_3(t)> \end{pmatrix}$$

$$= \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} \begin{pmatrix} |\nu_1(t)> \\ |\nu_2(t)> \\ |\nu_3(t)> \end{pmatrix}$$

where $H^{(m)}$ is the Hamiltonian in the mass representation, with eigen-values E_i

neutrino oscillations

Flavor eigenstates $|\nu_l >, |\nu_m >, |\nu_h >$

$$\begin{pmatrix} |\nu_l(t) > \\ |\nu_m(t) > \\ |\nu_h(t) > \end{pmatrix} = U \begin{pmatrix} |\nu_1(t) > \\ |\nu_2(t) > \\ |\nu_3(t) > \end{pmatrix}$$

with

$$\begin{aligned} U &= R_{23} R_{13} R_{12} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{\imath\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{\imath\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

s_{ij} (c_{ij}) reads for $\sin\theta_{ij}$ ($\cos\theta_{ij}$)

neutrino oscillations

Flavor conversion

$$P_{\alpha,\beta} = |<\nu_\alpha(t)|\nu_\beta(t=0)>|^2 \neq 0$$

then

$$\begin{aligned} P_{ll} &= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\left(\frac{\Delta m^2_{12} t}{4E_\nu}\right) - \sin^2 2\theta_{13}\sin^2\left(\frac{\Delta m^2_{31} t}{4E_\nu}\right) \\ &+ \sin^2 2\theta_{13}\sin^2\theta_{12}\left\{\sin^2\left(\frac{\Delta m^2_{31} t}{4E_\nu}\right) - \sin^2\left(\frac{\Delta m^2_{12} + \Delta m^2_{31}}{4E_\nu} t\right)\right\} \end{aligned}$$

where the mass differences are $\Delta m^2_{ij} = m^2_i - m^2_j$

neutrino oscillations

- mass hierarchy:
  - Normal: $m_1 \approx m_2 \ll m_3$
  - Inverse: $m_3 \ll m_1 \approx m_2$
  - Degenerate: $m_1 \approx m_2 \approx m_3$

where:

$$\begin{aligned} \Delta m_{13}^2 &\sim 10^{-3}\,\mathrm{eV}^2 \\ \Delta m_{12}^2 &\sim 10^{-5}\,\mathrm{eV}^2 \end{aligned}$$

neutrino mass scale

- double beta decay
  - The two neutrino mode has been observed (Sasha´s talk)
  - The neutrinoless mode of the double beta decay is still unobserved: it violates lepton number conservation and only lower limits of the half-lives have been determined (my talk in ANDES-I)
  - The observation will allow for a determination of the neutrino mass scale but still the mechanism may not be determined (next talk by Fedor)

active-sterile neutrino mixing

- From LSND (Large Scintillator Neutrino Detector) sterile neutrinos may be introduced
- The neutrino fields in the current

$$J_\mu \quad = \quad \overline{\Psi}\gamma_\mu \Psi$$

change to $\Psi_{ef} = \alpha\Psi_1 + \beta\Psi_s$, and the current reads

$$J_\mu \quad = \quad f\overline{\Psi}\gamma_\mu \Psi$$

- MiniBooNE has established stringent limits to the active-sterile neutrino mixing

active-sterile neutrino mixing

- KamLAND shows that the mixing (active-sterile) is rather weak
- WMAP measurements set-up a limit to the number of relativistic particles present at the time of BBN
- The square mass differences between active and sterile neutrinos may be of the order of 10^{-10} eV^2

active-sterile neutrino mixing

- active and sterile neutrinos (normal hierarchy)

The new state is degenerate with ν_1, then

$$
\begin{aligned}
\nu_1(t) &= \cos\phi\ \hat{\nu}_1(t) - \sin\phi\ \nu_s(t) \\
\nu_2(t) &= \hat{\nu}_2(t) \\
\nu_3(t) &= \hat{\nu}_3(t) \\
\nu_4(t) &= \sin\phi\ \hat{\nu}_1(t) + \cos\phi\ \nu_s(t)
\end{aligned}
$$

where ϕ is the new mixing angle

active-sterile neutrino mixing

$$\begin{pmatrix} \nu_l(t) \\ \nu_m(t) \\ \nu_h(t) \\ \nu_s(t) \end{pmatrix} = U \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \\ \nu_3(t) \\ \nu_4(t) \end{pmatrix}$$

with

$$U = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13} & 0 \\ -s_{12}c_{23} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} & 0 \\ s_{23}s_{12} - s_{13}c_{23}c_{12} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\phi & 0 & 0 & \sin\phi \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\phi & 0 & 0 & \cos\phi \end{pmatrix}$$

s_{ij} (c_{ij}) reads for $\sin\theta_{ij}$ ($\cos\theta_{ij}$)

active-sterile neutrino mixing

- active and sterile neutrinos in the inverse mass hierarchy

The new state is degenerate with ν_3

$$U = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13} & 0 \\ -s_{12}c_{23} - s_{23}s_{13}c_{12} & c_{23}c_{12} - s_{23}s_{13}s_{12} & s_{23}c_{13} & 0 \\ s_{23}s_{12} - s_{13}c_{23}c_{12} & -s_{23}c_{12} - s_{13}s_{12}c_{23} & c_{23}c_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\phi & \sin\phi \\ 0 & 0 & -\sin\phi & \cos\phi \end{pmatrix}$$

Sterile neutrinos in cosmology

- Problem: The observed abundance of $^7\mathrm{Li}$ does not coincide with the theoretical one if WMAP results are taken as input $\left((\Omega_B h^2)_{WMAP} = 0.0224 \pm 0.008\right)$

Abundance	Observed value	Theory
D	$(2.54 \pm 0.23) \times 10^{-5}$	$\left(2.57^{+0.17}_{-0.13}\right) \times 10^{-5}$
$^4\mathrm{He}$	0.2474 ± 0.0028	$0.2482^{+0.0003}_{-0.0004}$
$^7\mathrm{Li}$	$(1.26 \pm 0.26) \times 10^{-10}$	$(4.37 \pm 0.01) \times 10^{-10}$

- Hint: Standard BBN does not take oscillations into account explicitly
- It is then challenging to include them, as well as sterile neutrinos, and re-calculate BBN abundances

sterile neutrinos in cosmology

- Active and sterile neutrinos in the earlier Universe, by considering
  - two massive (free) neutrinos
  - two massive neutrinos interaction with electrons
  - three massive (active) neutrinos and one sterile neutrino
- Get new limits to reconciliate WMAP and BBN results

sterile neutrinos in cosmology

The steps are:

- Distribution functions for light neutrinos in presence of sterile neutrinos
- Reaction rates for the conversion of neutrons into protons (and viceversa)
- BBN abundances as a function of the mixing parameters, within WMAP limits
- Comparison with observed abundances

sterile neutrinos in cosmology

- Density matrix $\mathcal{F}_{ij} =< \nu_i | \nu_j >$, in the mass representation and for an expanding Universe is given by the equation:

$$\left(\frac{\partial \mathcal{F}}{\partial t} - \mathrm{H_H} E_\nu \frac{\partial \mathcal{F}}{\partial E_\nu} \right) \quad = \quad \imath \left[\mathcal{H}_0, \mathcal{F} \right]$$

where t is time, $\mathrm{H_H}$ is the expansion's constant $\mathrm{H_H} = \mu_P T^2$), E_ν is the energy of the neutrino and $\mathcal{H}_0 = \mathrm{diag}\left(E_1, E_2, E_3, E_4 \right)$

- Including the interaction between neutrinos and electrons

$$\left(\frac{\partial \mathcal{F}}{\partial t} - \mathrm{H_H} E_\nu \frac{\partial \mathcal{F}}{\partial E_\nu} \right) \quad = \quad \imath \left[\mathcal{H}_0 + \sqrt{2} G_F \left(n_e(T) - \frac{8}{3 M_W^2} \rho_e(T) E_\nu \right) A, \mathcal{F} \right]$$

sterile neutrinos in cosmology

- Adding one sterile neutrino one the new density matrix

$\mathcal{F}_{ij} =< \nu_i|\nu_j >$, (mass basis, expanding universe)

$$\left(\frac{\partial \mathcal{F}}{\partial t} - \mathrm{H_H} E_\nu \frac{\partial \mathcal{F}}{\partial E_\nu}\right) \quad = \quad \imath\left[\mathcal{H}_0, \mathcal{F}\right]$$

$\mathcal{H}_0 = \mathrm{diag}\left(E_1, E_2, E_3, E_4\right)$

- Initial condition $\left(f_{ij} = U\mathcal{F}U^\dagger\Big|_{ij}\right)$:

$$\left.\begin{pmatrix} f_a & f_{as} \\ f_{sa} & f_s \end{pmatrix}\right|_{T_0} \quad = \quad \frac{1}{1+e^{E_\nu/T_0}} \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

sterile neutrinos in cosmology

- Solutions in the flavor basis:
  - two states:

$$f_l \quad = \quad \frac{1}{1+e^{E_\nu/T}}\left\{1+\frac{\sin^2 2\phi}{2}\left[\cos\left(\frac{\Delta m^2}{6\mu_P}\frac{T}{E_\nu}\left(\frac{1}{T^3}-\frac{1}{T_0^3}\right)\right)-1\right]\right\}$$

  - three states (normal hierarchy):

$$f_l = \frac{1}{1+e^{E_\nu/T}} + \frac{\cos^2\theta_{13}\cos^2\theta_{12}}{1+e^{E_\nu/T}}\frac{\sin^2 2\phi}{2}\left[\cos\left(\frac{\Delta m^2}{6\mu_P}\frac{T}{E_\nu}\left(\frac{1}{T^3}-\frac{1}{T_0^3}\right)\right)-1\right]$$

  - three states (inverse hierarchy):

$$f_l \quad = \quad \frac{1}{1+e^{E_\nu/T}}\left\{1+\sin^2\theta_{13}\frac{\sin^2 2\phi}{2}\left[\cos\left(\frac{\Delta m^2}{6\mu_P}\frac{T}{E_\nu}\left(\frac{1}{T^3}-\frac{1}{T_0^3}\right)\right)-1\right]\right\}$$

BBN and reactions

- reaction rates for the $n \leftrightarrow p$ process are written:

$$\begin{aligned}
\lambda_{(\nu+n\to p+e^-)} &= \kappa \int_0^\infty dp_\nu \; p_\nu E_\nu p_e E_e \left(1 - f_e\right) f_l \\
\lambda_{(e^+ + n\to p+\overline{\nu})} &= \kappa \int_0^\infty dp_e \; p_\nu E_\nu p_e E_e \left(1 - f_l\right) f_e \\
f_e &= \left(1 + e^{E_e/T}\right)^{-1} \\
f_l &= \left(1 + e^{E_\nu/T}\right)^{-1} \left\{1 - \frac{\sin^2 2\phi}{2}\xi\left[1 - g\left(\Delta m^2, E_\nu, T\right)\right]\right\}
\end{aligned}$$

BBN

- Primordial elements:

$$\frac{dY_i}{dt} = J(t) - \Gamma(t) Y_i$$

$J(t)$ and $\Gamma(t)$ are the source terms, Y_i is the abundance of the element i

- Quality of the fit

$$\chi^2 = \sum \frac{\left(Y_x^{obs} - Y_x^{teo}\left(\Omega_B h^2, \sin^2 2\phi, \Delta m^2\right)\right)^2}{\sigma_x^2}$$

BBN (results)

- Two active neutrinos

Δm^2 [eV^2]	All data		excluding ^{7}Li	
	$\sin^2 2\phi \pm \sigma$	$\Omega_B h^2 \pm \sigma$	$\sin^2 2\phi \pm \sigma$	$\Omega_B h^2 \pm \sigma$
10^{-6}	0.002 ± 0.078	$0.025^{+0.002}_{-0.001}$	$0.220^{+0.094}_{-0.086}$	0.023 ± 0.002
10^{-8}	0.002 ± 0.033	0.025 ± 0.001	$0.221^{+0.095}_{-0.092}$	0.023 ± 0.002
10^{-10}	0.002 ± 0.078	$0.025^{+0.002}_{-0.001}$	$0.213^{+0.094}_{-0.086}$	0.023 ± 0.002

- Three active neutrinos:

All data		excluding ^{7}Li	
$\sin^2 2\phi \pm \sigma$	$\Omega_B h^2 \pm \sigma$	$\sin^2 2\phi \pm \sigma$	$\Omega_B h^2 \pm \sigma$
0.000 ± 0.026	0.0253 ± 0.0015	0.018 ± 0.098	0.0216 ± 0.0017

- Fixed baryon density (two sterile neutrinos)

Group I			
Data	$\phi_1 \pm \sigma$	$\phi_2 \pm \sigma$	$\chi^2/(N-2)$
D + ^{4}He + ^{7}Li	0.014 ± 0.220	0.063 ± 0.181	6.28
D + ^{4}He	0.000 ± 0.165	0.000 ± 0.165	0.93
Group II			
Data	$\phi_1 \pm \sigma$	$\phi_2 \pm \sigma$	$\chi^2/(N-2)$
D + ^{4}He + ^{7}Li	0.016 ± 0.079	0.016 ± 0.079	7.02
D + ^{4}He	0.016 ± 0.063	0.016 ± 0.063	1.79
Group III			
Data	$\phi_1 \pm \sigma$	$\phi_2 \pm \sigma$	$\chi^2/(N-2)$
D + ^{4}He + ^{7}Li	0.016 ± 0.047	0.016 ± 0.047	6.50
D + ^{4}He	0.000 ± 0.055	0.000 ± 0.055	1.90

• Variable baryon density (two sterile neutrinos). Best-fit parameter values and 1σ errors, considering the mean value of the oscillating terms that include the mass split. The baryon-to-photon ratio η_B is in units of 10^{-10}.

Group I				
Data	$\eta_B \pm \sigma$	$\phi_1 \pm \sigma$	$\phi_2 \pm \sigma$	$\frac{\chi^2}{N-3}$
D+^{4}He+^{7}Li	5.09 ± 0.18	0.01 ± 0.13	0.01 ± 0.13	2.55
D+^{4}He	$5.85^{+0.31}_{-0.29}$	0.00 ± 0.16	0.00 ± 0.16	0.93
Group II				
Data	$\eta_B \pm \sigma$	$\phi_1 \pm \sigma$	$\phi_2 \pm \sigma$	$\frac{\chi^2}{N-3}$
D+^{4}He+^{7}Li	5.09 ± 0.17	0.02 ± 0.05	0.02 ± 0.05	3.58
D+^{4}He	$6.05^{+0.22}_{-0.31}$	0.01 ± 0.06	0.01 ± 0.06	1.93
Group III				
Data	$\eta_B \pm \sigma$	$\phi_1 \pm \sigma$	$\phi_2 \pm \sigma$	$\frac{\chi^2}{N-3}$
D+^{4}He+^{7}Li	5.27 ± 0.27	0.01 ± 0.05	0.01 ± 0.05	3.62
D+^{4}He	$6.05^{+0.32}_{-0.31}$	0.01 ± 0.05	0.01 ± 0.06	2.05

• Best-fit parameter values and 1σ errors, considering the mean value of the oscillating terms that include the mass split and with the inclusion of only one extra neutrino.

Group I			
Data	$\eta_B \pm \sigma$	$\phi_1 \pm \sigma$	$\frac{\chi^2}{N-2}$
D+^{4}He+^{7}Li	5.09 ± 0.12	0.030 ± 0.110	2.39
D+^{4}He	5.85 ± 0.27	0.010 ± 0.144	0.85
Group II			
Data	$\eta_B \pm \sigma$	$\phi_1 \pm \sigma$	$\frac{\chi^2}{N-2}$
D+^{4}He+^{7}Li	5.21 ± 0.23	0.035 ± 0.050	3.34
D+^{4}He	5.98 ± 0.28	0.015 ± 0.055	1.77
Group III			
Data	$\eta_B \pm \sigma$	$\phi_1 \pm \sigma$	$\frac{\chi^2}{N-2}$
D+^{4}He+^{7}Li	5.12 ± 0.13	0.035 ± 0.033	3.84
D+^{4}He	5.98 ± 0.28	0.015 ± 0.048	2.23

Neutrinos from exotic objects

• We shall briefly discuss the possibility of detecting neutrinos from micro quasars:

- in presence of neutrino oscillations
- by considering the interaction of the neutrinos with matter
- by getting the signal to noise ratios, as a function of the size of the detector and of the observation time

neutrinos form micro-quasars

- Element of the calculations
  - neutrino oscillations
  - interaction of the neutrinos with matter (MSW effect)
  - Folding of emission and detection processes

neutrinos form micro-quasars

- Proton spectra

$$
\begin{aligned}
p + p &\rightarrow p + p + \xi_{\pi^0}\pi^0 + \xi_\pi(\pi^+ + \pi^-) \\
q_\gamma(\psi, E_\gamma, z, \theta) &= 4\pi\eta_A\sigma_{pp}(E_p)\frac{2Z^{(\alpha)}_{p\rightarrow\pi^0}}{\alpha}J_p(\psi, E_\gamma, z, \theta)
\end{aligned}
$$

(1)

neutrinos form micro-quasars

- Muon-neutrino intensity

$$I_\nu(E_\nu, \psi, \theta) = 4 \int \mathrm{d}V \, \frac{f_p}{m_p} \rho_w(r_w) q_\gamma(\psi, 2E_\nu, z, \theta)$$

$$S_\nu(\theta) = \frac{T_{obs} A_{eff}}{4\pi d^2} \int_0^{2\pi} \mathrm{d}\psi \int_{10^6\,\mathrm{MeV}}^{E_\nu^{max}} I_\nu(E, \psi, \theta) P(E) \mathrm{d}E$$

(2)

neutrinos form micro-quasars

- The noise above 1 TeV

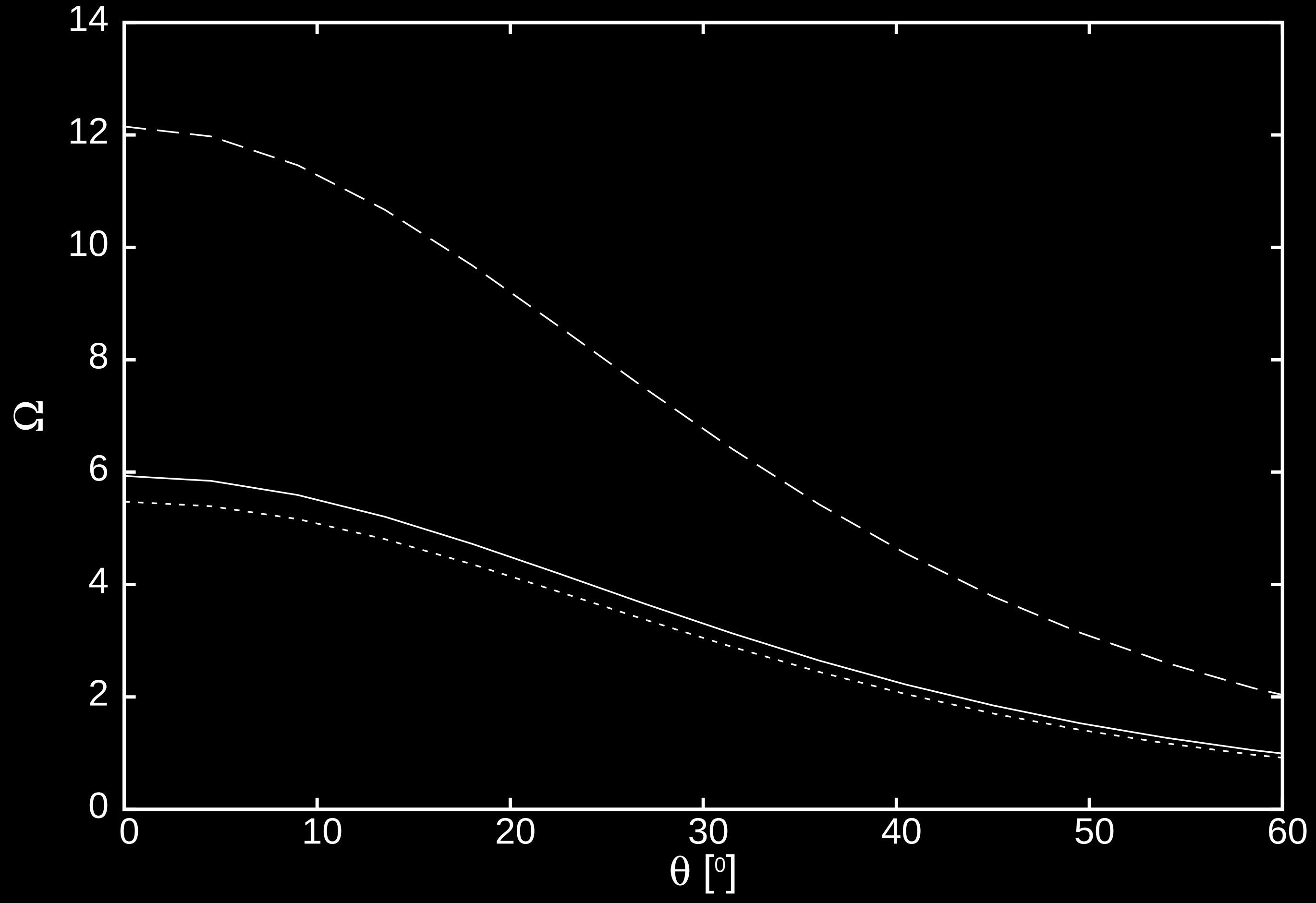
$$N = \sqrt{T_{obs} A_{eff} \Delta\Omega \int_{10^6 \mathrm{MeV}}^{E_\nu^{max}} F_B(E) P(E) \mathrm{d}E}$$

where

$$F_B(E) = 2 \left(\frac{E}{10^3 \mathrm{MeV}} \right)^{-3.21} \mathrm{MeV}^{-1}\, \mathrm{m}^{-2}\, \mathrm{s}^{-1}\, \mathrm{sr}^{-1}$$

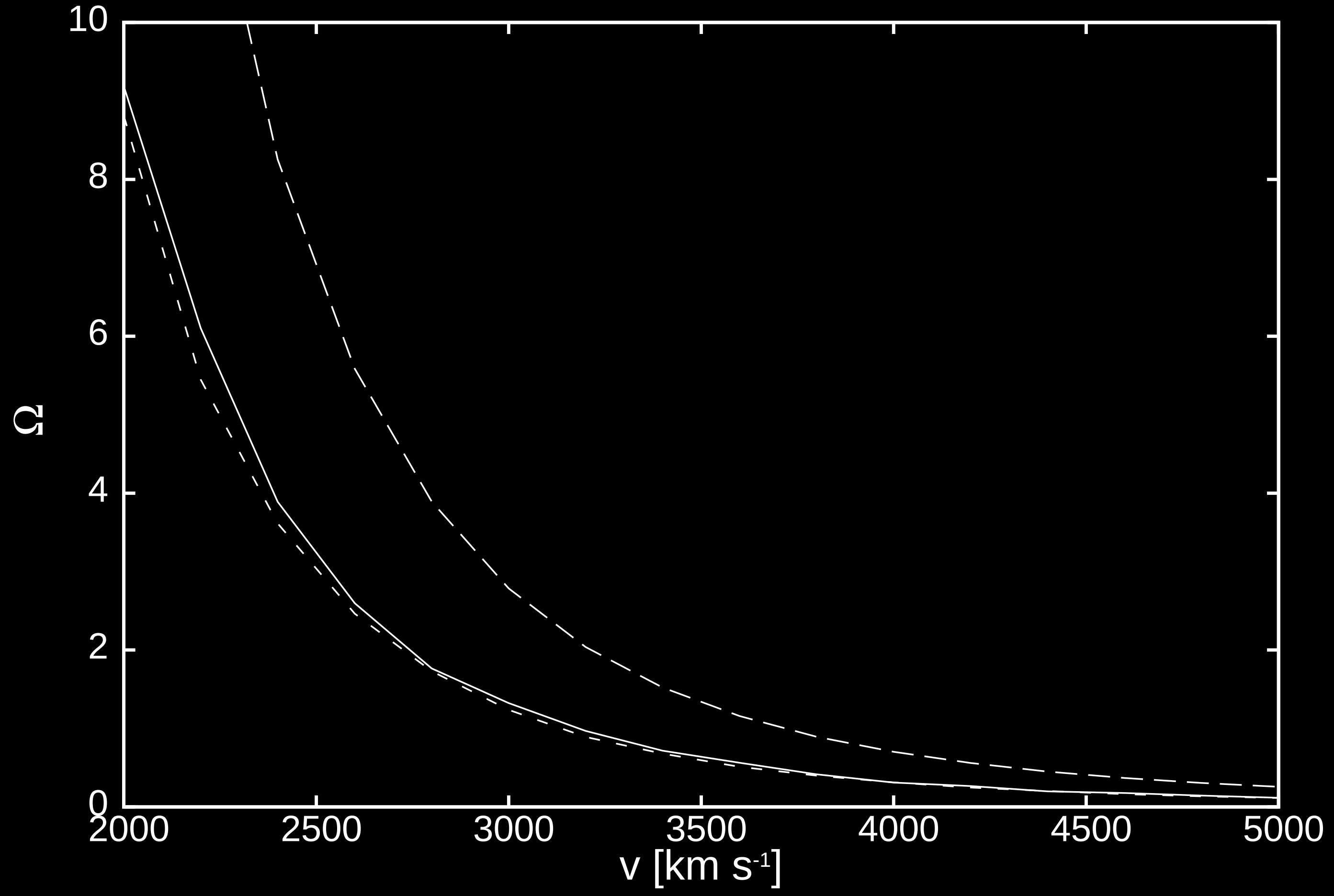
neutrinos form micro-quasars

- Neutrino signal-to-noise ratio as a function of the viewing angle θ.



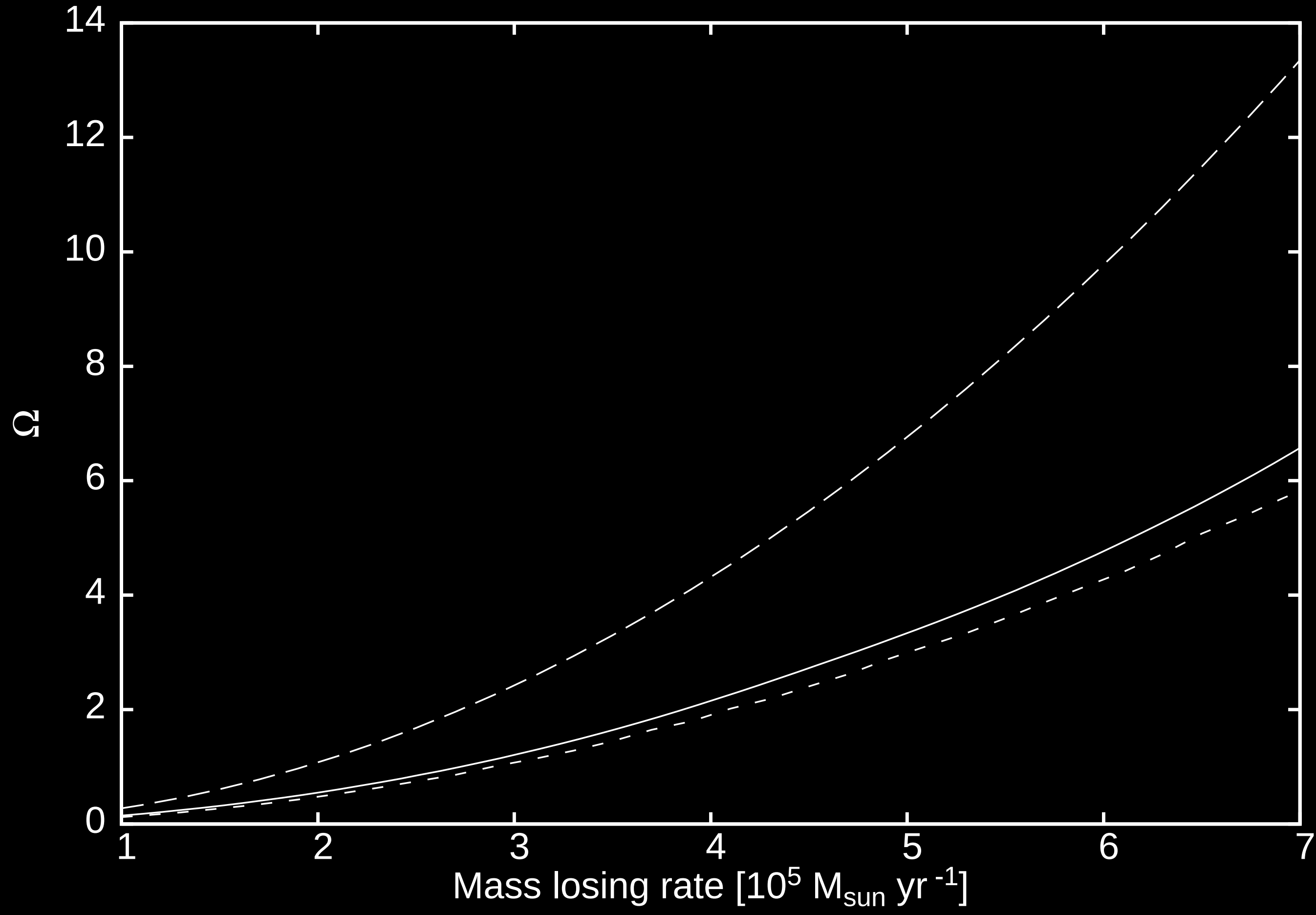
neutrinos form micro-quasars

- Neutrino signal-to-noise ratio as a function of v_∞.



neutrinos form micro-quasars

- Neutrino signal-to-noise ratio as a function of $\dot{M}_\star$.



Summary

• The analysis of the compatibility between theoretical and observed BBN abundances shows that:

- BBN is indeed sensitive to the neutrino mass hierarchy oscillation (and mixing) parameters
- the baryon density and the mixing angle (one sterile neutrino scenario), $\Omega_B h^2$ y $\sin^2 2\phi$, agree with WMAP and LSND if the data on $^7\mathrm{Li}$ are excluded
- The same feature reveals for the mixing between three active neutrinos and two sterile neutrinos